

10.4

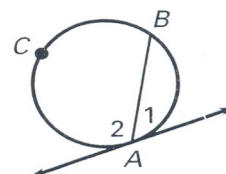
Other Angle Relationships in Circles

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- Goals**
- Use angles formed by tangents and chords to solve problems in geometry.
 - Use angles formed by lines that intersect a circle to solve problems.

THEOREM 10.12

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



$$m\angle 1 = \frac{1}{2} \text{ _____}, \quad m\angle 2 = \frac{1}{2} \text{ _____}$$

Example 1 Finding an Angle Measure

In the diagram below, \overleftrightarrow{KL} is tangent to the circle. Find $m\angle KLM$.

$$m\angle KLM = \frac{1}{2} m\widehat{MJL}$$

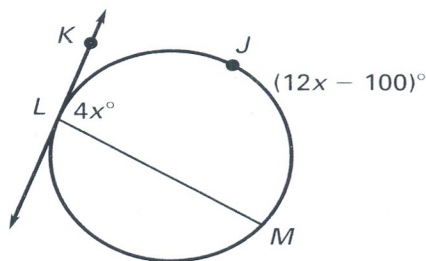
$$4x = \frac{1}{2} (12x - 100)$$

$$8x = 12x - 100$$

$$100 = 4x$$

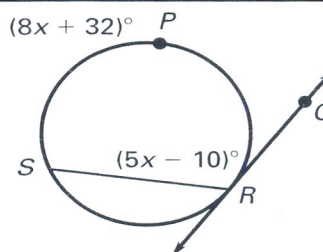
$$25 = x$$

$$\text{Answer } m\angle KLM = \left(\frac{1}{2} \cdot 100 \right)^\circ = 50^\circ$$



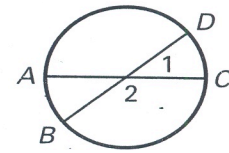
Checkpoint Complete the following exercise.

1. \overleftrightarrow{QR} is tangent to the circle.
Find $m\angle QRS$.



THEOREM 10.13

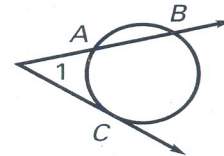
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the _____ of the measures of the arcs intercepted by the angle and its vertical angle.



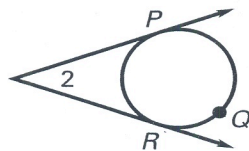
$$m\angle 1 = \frac{1}{2}(m______ + m______), m\angle 2 = \frac{1}{2}(m______ + m______)$$

THEOREM 10.14

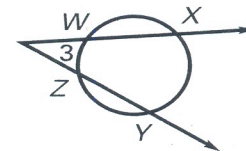
If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the _____ of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(m______ - m______)$$



$$m\angle 2 = \frac{1}{2}(m______ - m______)$$



$$m\angle 3 = \frac{1}{2}(m______ - m______)$$

Example 2 *Measure of an Angle Formed by Two Chords*

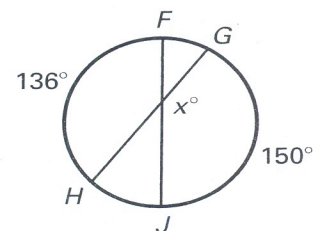
Find the value of x .

Solution

$$x^\circ = \frac{1}{2}(m______ + m______) \quad \text{Apply Theorem 10.13.}$$

$$x^\circ = \frac{1}{2}(______^\circ + ______^\circ) \quad \text{Substitute.}$$

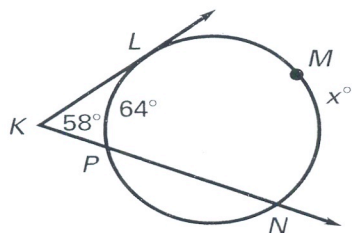
$$x = ______ \quad \text{Simplify.}$$



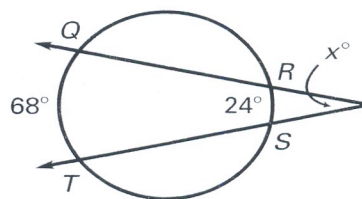
Example 3 Using Theorem 10.14

Find the value of x .

a.



b.



Solution

a. $m\angle LKP = \frac{1}{2}(m\widehat{LMN} - m\widehat{LP})$

Apply Theorem 10.14.

$\text{---}^\circ = \frac{1}{2}(x^\circ - \text{---}^\circ)$

Substitute.

$\text{---} = x - \text{---}$

Multiply each side by 2.

$\text{---} = x$

Solve for x .

b. $x = \frac{1}{2}(m\widehat{QT} - m\widehat{RS})$

Apply Theorem 10.14.

$= \frac{1}{2}(\text{---} - \text{---})$

Substitute.

$= \frac{1}{2}(\text{---})$

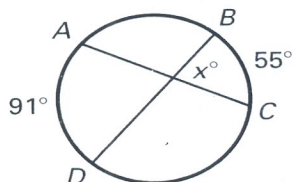
Subtract.

$= \text{---}$

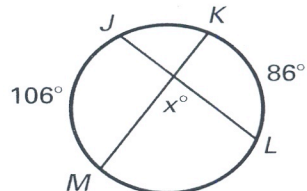
Multiply.

Checkpoint Find the value of x .

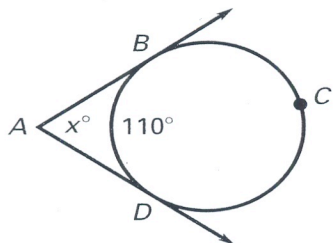
2.



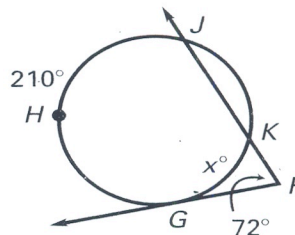
3.



4.



5.



Study Guide

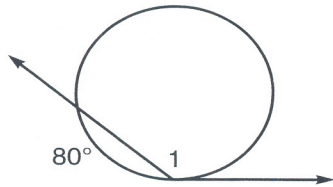
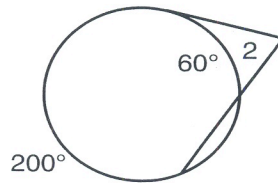
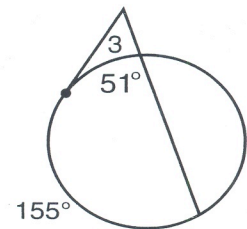
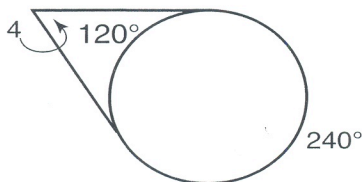
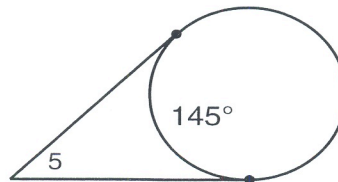
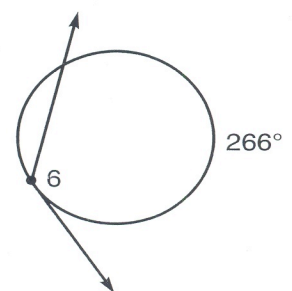
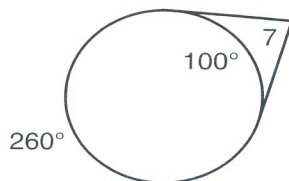
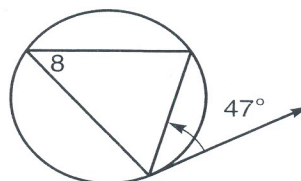
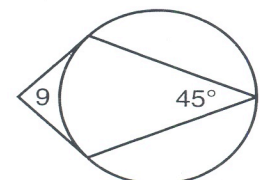
10.4 Blue

Secant-Tangent Angles

You can find the measures of angles formed by secants and tangents by using the following theorems.

- If a secant-tangent angle has its vertex outside a circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs.
- If a secant-tangent angle has its vertex on a circle, then its degree measure is one-half the degree measure of the intercepted arc.
- The degree measure of a tangent-tangent angle is one-half the difference of the degree measures of the intercepted arcs.

Find the measure of each angle. Assume segments that appear to be tangent are tangent.

1. $m\angle 1$ 2. $m\angle 2$ 3. $m\angle 3$ 4. $m\angle 4$ 5. $m\angle 5$ 6. $m\angle 6$ 7. $m\angle 7$ 8. $m\angle 8$ 9. $m\angle 9$ 

Skills Practice

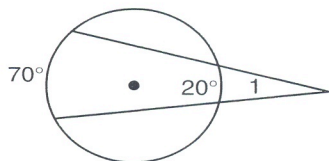
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10.4 Blue

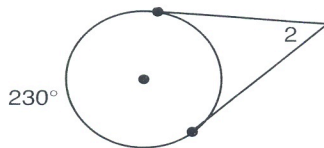
Secant-Tangent Angles

Find the measure of each angle. Assume segments that appear to be tangent are tangent.

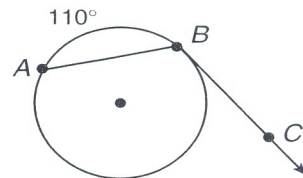
1. $\angle 1$



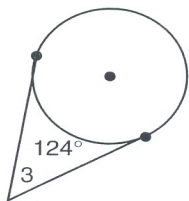
2. $\angle 2$



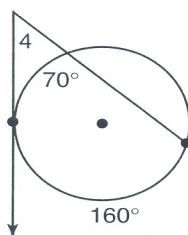
3. $\angle ABC$



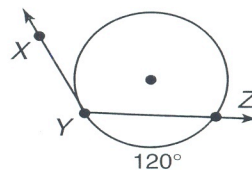
4. $\angle 3$



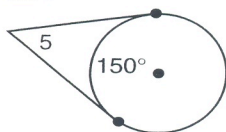
5. $\angle 4$



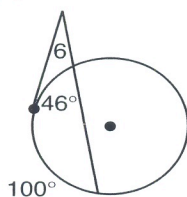
6. $\angle XYZ$



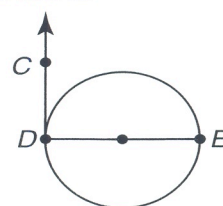
7. $\angle 5$



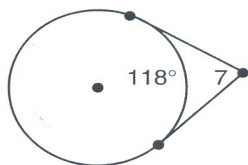
8. $\angle 6$



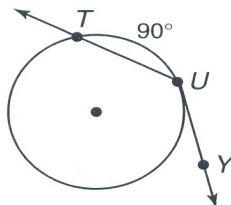
9. $\angle CDE$



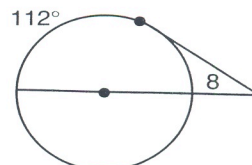
10. $\angle 7$



11. $\angle TUY$



12. $\angle 8$



Study Guide

1

Secant Angles

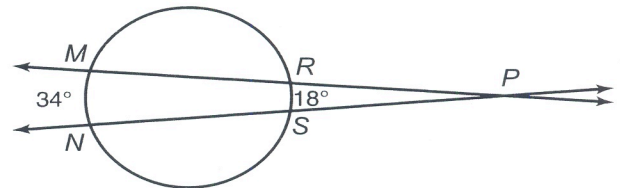
A line that intersects a circle in exactly two points is called a **secant** of the circle. You can find the measures of angles formed by secants and tangents by using the following theorems.

- If a secant angle has its vertex inside a circle, then its degree measure is one-half the sum of the degree measures of the arcs intercepted by the angle and its vertical angle.
- If a secant angle has its vertex outside a circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs.

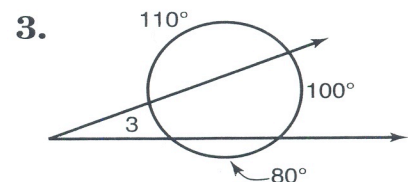
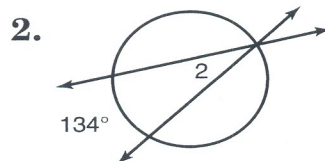
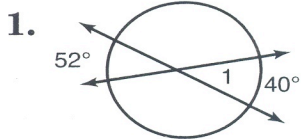
Example: Find the measure of $\angle MPN$.

You can use the last theorem above.

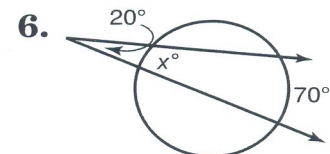
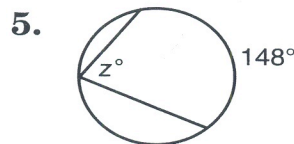
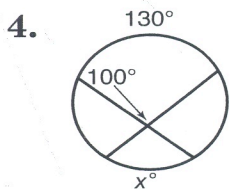
$$\begin{aligned} m\angle MPN &= \frac{1}{2}(m\widehat{MN} - m\widehat{RS}) \\ &= \frac{1}{2}(34 - 18) \\ &= \frac{1}{2}(16) \text{ or } 8 \end{aligned}$$



Find the measure of each numbered angle.



In each circle, find the value of x .



Skills Practice

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Secant Angles

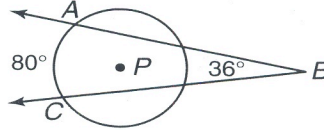
Find each measure.

10.4 Blue

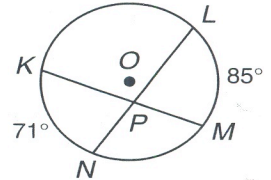
1. $m\angle EFB$



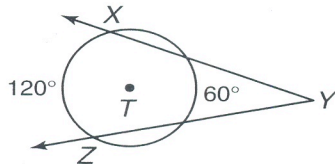
2. $m\angle ABC$



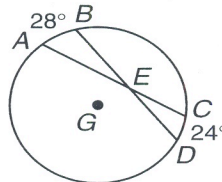
3. $m\angle LPM$



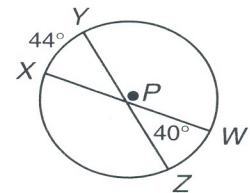
4. $m\angle XYZ$



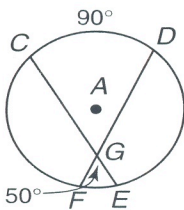
5. $m\angle CED$



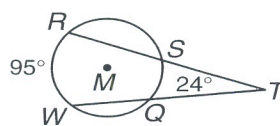
6. $m\widehat{WZ}$



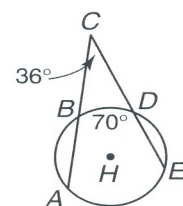
7. $m\widehat{FE}$



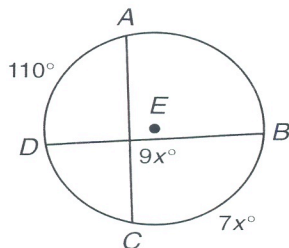
8. $m\widehat{SQ}$



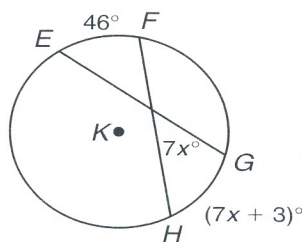
9. $m\widehat{AE}$


Find the value of x in each circle. Then find the given measure.

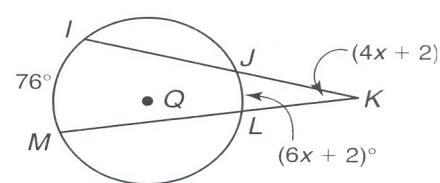
10. $m\widehat{BC}$



11. $m\widehat{HG}$



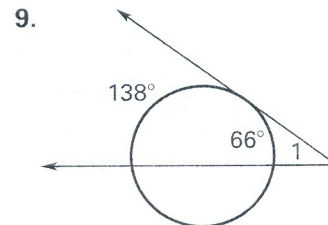
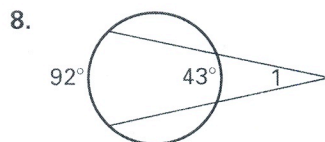
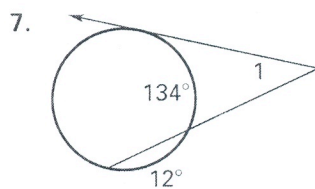
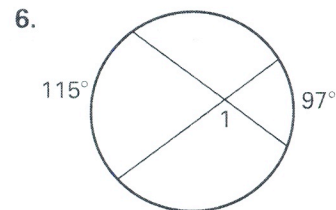
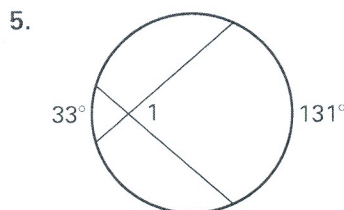
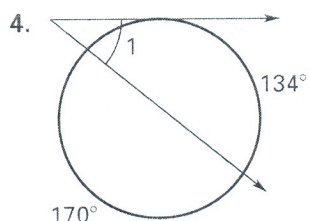
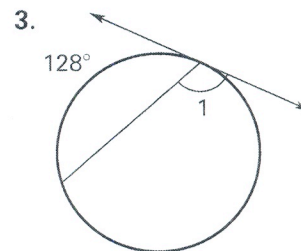
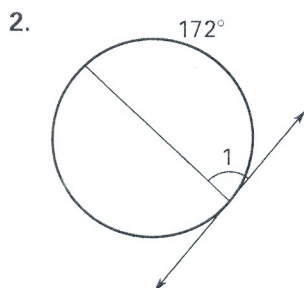
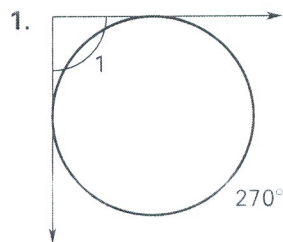
12. $m\widehat{JL}$



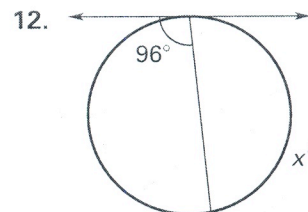
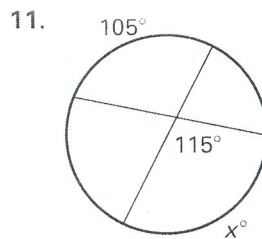
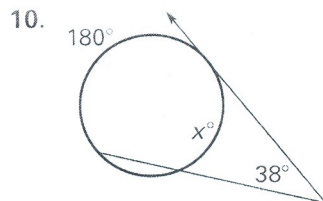
Practice A

For use with pages 621–627

Find the measure of $\angle 1$.



Write an equation that can be used to solve for x . Then solve the equation for x .



Use the diagram of $\odot A$ to write the $m\angle 1$, $m\angle 2$, and $m\angle 3$ in order of increasing measure.

